647510 **BSc4MatC0401** **Seat No :________**

B.Sc. Semester - 4 (*CBCS*) Examination
July-2025 [OLD COURSE]
MATHEMATICS(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Attempt any one (07)

(1) State and prove necessary and sufficient condition for a non-empty Subset W of a vector space V to be a subspace of V.

(2) Prove that set of vectors $\{\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_n\}$ is L.D. iff there exist vector $\bar{v}_k$, $2 \le k \le n$, which is linear combination of its preceding vectors $\bar{v}_1, \bar{v}_2, \ldots \bar{v}_{k-1}$.

Que.1(B) Attempt any one (04)

(1) Check whether vector $(0,-1, 1) \in$ SPA. where set A=$\{(2,1,0),(-1,0,1),(0,1,2)\}$.

(2) Check whether set A= {sin(x), cos(x), cos(x-$\frac{\pi}{4}$)} of real vector space $C[0,2\pi]$ is linearly dependent or linearly independent.

Que.1(C) Attempt any one (03)

(1) Prove that any superset of linearly dependent set is also linearly dependent.

(2) Check whether set W = $\{(x, y, z) / x + y + z = 0, x, y, z \in R\}$ is subspace of R^3.

Que.2(A) Attempt any one (07)

(1) Prove that there exists a basis for each finite dimensional vector space V.

(2) State and prove extension theorem for basis of vector space.

Que.2(B) Attempt any one (04)

(1) Extend set A= {(1, 1, 1),(2,0,0)} of vector space R^3 to form a base of R^3.

(2) Prove that set $\{1-x, 1+x, 1-x^2\}$ is base of $P_2(R)$. Find coordinates of $3+7x+2x^2$ with respect to this base.

Que.2(C) Attempt any one (03)

(1) W is subspace of finite dimensional vector space V then dim W $\le$ dim V.

(2) Prove that the intersection of two subspaces of a vector space V is also a subspace of V.

Que.3(A) Attempt any one (07)

(1) Let L (U, V) denote the set of all linear transformations from a vector space U to a vector space V. Prove that L(U, V) is a vector space over R under usual operations.

(2) State and prove rank – nullity theorem for linear transformation.

Que.3(B) Attempt any one (04)

(1) Prove that a linear transformation T: U $\rightarrow$ V is one-one iff $N_T = \{\theta\}$, where N_T denotes kernel of T and θ denote zero vector of U.

(2) Let T: $R^3 \rightarrow R^3$ defined by T(x, y, z) = (x - y + z, y - z, z) then find T^{-1}.

Que.3(C) Attempt any one (03)

(1) Define: Linear transformation, Range of linear transformation, Nilpotent linear transformation.

(2) Let T: U $\rightarrow$ V be a linear transformation and N_T denote kernel of T. Prove that N_T is a subspace of U.

Que.4(A) Attempt any one (07)

(1) Let T: V → V be a linear transformation and let B be any basis of V. Then, T is singular iff det([T; B]) = 0.

(2) Find eigenvalues and eigenvectors of the linear transformation T: $R^3 \rightarrow R^3$ defined by T(x, y, z) = (x + y + z, x + y + z, x + y + z)

Que.4(B) Attempt any one (04)

(1) Let T: $R^3 \rightarrow R^2$ be a linear transformation defined by T(x, y, z) = (x + y, y + z). Find [T: B_1, B_2], where B_1= {(1, 1, 1), (1, 0, 0), (1, 1, 0)} and B_2 = {e_1, e_2} are basis of R^3 and R^2 respectively.

(2) In usual notation prove that R_T is a subspace of V.

Que.4(C) Attempt any one (03)

(1) Define: Eigen value of a linear transformation, Eigen vector of a linear transformation, Eigen basis of a linear transformation.

(2) Find Eigen values of the linear transformation T: $R^2 \rightarrow R^2$; T (a, b) = (3b, 2a - b).

Que.5(A) Attempt any one (07)

(1) Find radius of curvature for Cartesian curves.

(2) Find the position and nature of double points of the curve

$$x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0$$

Que.5(B) Attempt any one (04)

(1) Using Newton's method, find the radius of curve

$$x^4 + y^4 + x^3 - y^3 + x^2 - y^2 + y = 0$$

(2) Find the radius of curvature at any point of $x = a(\theta + sin\theta); y = a(1 - cos\theta)$

Que.5(C) Attempt any one (03)

(1) Show that the curve given by y = log x is concave downwards

(2) Find asymptotes of the curve $(x^2 + y^2)x - ay^2 = 0$ parallel to coordinate axes.

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